Quantification:
Quantifiers and the Rest of the Sentence

1. Introduction.

- We have seen that Determiners express a relation between two sets of individuals A and B. Set A is provided by the N’ sister of the Determiner in the syntactic tree, and set B is provided by the rest of the sentence, as in (1).

(1) \[
\text{Every} \quad [\text{American city}]_N' \quad \downarrow \quad [\text{is near the Atlantic Ocean}]_B \\
\{x: x \text{ is American in w and } x \text{ is a city in w} \} \quad \{x: x \text{ is near the A. O. in w} \}
\]

- When the Quantificational NP (QuNP) is in subject position, it is very easy to give rules to interpret the sentence:

\[
\begin{align*}
(3) & \quad \lbrack \text{every } N' \rbrack^w \wedge \lbrack \text{VP} \rbrack^w = 1 \quad \text{iff} \quad \lbrack N' \rbrack^w \subseteq \lbrack \text{VP} \rbrack^w \\
(4) & \quad \lbrack \text{some } N' \rbrack^w \wedge \lbrack \text{VP} \rbrack^w = 1 \quad \text{iff} \quad \lbrack N' \rbrack^w \cap \lbrack \text{VP} \rbrack^w \neq \emptyset \\
(5) & \quad \lbrack \text{no } N' \rbrack^w \wedge \lbrack \text{VP} \rbrack^w = 1 \quad \text{iff} \quad \lbrack N' \rbrack^w \cap \lbrack \text{VP} \rbrack^w = \emptyset \\
(6) & \quad \lbrack \text{at most four } N' \rbrack^w \wedge \lbrack \text{VP} \rbrack^w = 1 \quad \text{iff} \quad | \lbrack N' \rbrack^w \cap \lbrack \text{VP} \rbrack^w | \leq 4 \\
(7) & \quad \lbrack \text{most } N' \rbrack^w \wedge \lbrack \text{VP} \rbrack^w = 1 \quad \text{iff} \quad | \lbrack N' \rbrack^w \cap \lbrack \text{VP} \rbrack^w | \geq 2/3 \cdot | \lbrack N' \rbrack^w | \\
(8) & \quad \lbrack \text{many}_{\text{prop}} N' \rbrack^w \wedge \lbrack \text{VP} \rbrack^w = 1 \quad \text{iff} \quad | \lbrack N' \rbrack^w \cap \lbrack \text{VP} \rbrack^w | > 1/2 \cdot | \lbrack N' \rbrack^w | \\
(9) & \quad \lbrack \text{many}_{\text{card}} N' \rbrack^w \wedge \lbrack \text{VP} \rbrack^w = 1 \quad \text{iff} \quad | \lbrack N' \rbrack^w \cap \lbrack \text{VP} \rbrack^w | > n
\end{align*}
\]

\[\text{Each of the semantic rules in (3)-(9) could be of course replaced by the bare lexical entry for the quantifier, as in (3') below, coupled with the general Determiner interpretation rules in (i)-(ii). We do it in terms of rules for simplicity, and because the book formulates the final solution as a rule and not as a lexical entry.}\]

\[
(3') \quad \lbrack \text{every} \rbrack^w = \{<A,B>: A \subseteq B\} \\
(i) \quad \lbrack \text{every } N'_\text{QuNP} \rbrack^w = \{B: \lbrack N' \rbrack^w \subseteq B\} \\
(ii) \quad \lbrack \text{QuNP} \text{ VP} \rbrack^w = 1 \quad \text{iff} \quad \lbrack \text{VP} \rbrack^w \in \lbrack \text{QuNP} \rbrack^w
\]
But what happens when the QuNP is not in subject position?

(10) [John admires] every [American city]_N' 

\[\downarrow\]

Set B 
\{x: x is such that …??…\} 

Set A 
\{x: x is American in w and x is a city in w\}

\[\downarrow\]

(11) \[S\]

q p

NP VP

g e i

John admires NP

g 6 e i

every American city

The problem, and two lines to tackle it:

(12) The problem: 

How can we retrieve set B from the syntactic tree of a sentence with a QuNP in non-subject position?

(13) Two main possible lines:

A. Enrich the semantic rules, with a different semantic rule for each quantifier for each level of syntactic embedding.

B. Enrich the syntactic rules, so that syntactic operations prior to interpretation deliver a syntactic structure easier to interpret.

2. Line A: Enriching the semantic rules.

Besides the rules in (3)-(9) to interpret QuNPs in subject position (i.e., in first level of embedding position), we would have rules for QuNPs in second level of embedding, third level of embedding, etc.

Rules for QuNPs in SECOND LEVEL of embedding:

(14) \[\llbracket \text{NP} \ [V_{tr} \ [\text{every N'}]] \rrbracket^w = 1 \text{ iff } \llbracket \text{N'} \rrbracket^w \subseteq \{x: <\llbracket \text{NP} \rrbracket^w, x_0 > \in \llbracket V_{tr} \rrbracket^w\}\]

QUESTION: Spell out the rules for some, at most four and most in second level of embedding:

(15) \[\llbracket \text{NP} \ [V_{tr} \ [\text{some N'}]] \rrbracket^w = 1 \text{ iff }\]

(16) \[\llbracket \text{NP} \ [V_{tr} \ [\text{at most four N'}]] \rrbracket^w = 1 \text{ iff }\]

(17) \[\llbracket \text{NP} \ [V_{tr} \ [\text{most N'}]] \rrbracket^w = 1 \text{ iff }\]
Rules for QuNPs in third level of embedding:

(18) John showed every American city to Kate.

(19) S
    w          Vp
    o
    NPsu    e  i
    e  i
    e  i
    V_ditr  NpDo

Exercise (to be handed in!!!): Assuming the syntactic tree in (19) for ditransitive structures, spell out the semantic rules to interpret every, no, manyprop and manycard in third level of embedding position:

(19) [[NP [ [V_ditr [every N'] ] NpI0] ]]w = 1  
    iff …

(20) [[NP [ [V_ditr [no N'] ] NpI0] ]]w = 1  
    iff …

(21) [[NP [ [V_ditr [manyprop N'] ] NpI0] ]]w = 1  
    iff …

(22) [[NP [ [V_ditr [manycard N'] ] NpI0] ]]w = 1  
    iff …

A serious drawback of line A:
The number of semantic rules explodes. We saw in part I of the course that we need different semantic rules for combining a verb with a referential NP (e.g. a proper name) at each level of embedding. For Determiners, we would need not only a different semantic rule for each level of embedding, but a different semantic rule for each level-of-embedding \times scopal relation combination. E.g., we would need one rule for everylevel-narrow to generate reading (23a) and one for everylevel-wide to generate reading (23b).

(23) A student admires every professor.
    a. some >> every: ‘There is a student x such that, for every professor y, x admires y.’
    b. every >> some: ‘For every professor y, there is a (possibly different) student x such that x admires y.’

Note: In Take Home II, you will be asked to give the semantic rule for everylevel-narrow and everylevel-narrow. You should start thinking about the answer as we proceed through line B.
3. Line B: Enriching the syntactic rules before interpretation.

- The “Y” model of NatLg grammar (Chomsky 1965):

(24) The “Y” model:

```
   Deep Structure (D-Str)
   g
   Surface Structure (S-Str)
   q           p
```

Phonological Form (PF) Logical Form (LF)

- Phrase structure rules and transformations.

Phrase structures rules like the ones in (25) produce the syntactic level of representation called ‘Deep Structure’. After that, transformations of the type “move α” may apply, yielding e.g. the topicalization in (27).

(25) Phrase structure rules:

```
S  →  NP  VP
NP →  Npr
NP →  Det  N'
...```

(26) Transformations: Topicalization.

```
[S X NP Y]  ⇒  [s NP, X e Y]
```

(27) Beans, I like.

i. Phrase structure rules:  
[s I like beans]  
ii. Transformation:  
[s I like beans] ⇒  [s Beans, I like e ]

- On transformations: where exactly they apply, and their scopal properties.

  - Some transformations, e.g., moving a wh-phrase, have an effect on the scope of the moved element. For example, we can apply “move wh” on the D-Str in (28i) to obtain an S-Str where the wh-phrase which of these books is at the top of the local S, or to obtain an S-Str where it is at the top of the matrix S. Each of the two sentences has a different scopal reading.

(28)  
i. D-Str:  [s John knows [s Mary bought which of these books] ]

ii. S-Str:  John knows [s which of these books Mary bought e ]

iii. S-Str:  [s Which of these books (does) John know Mary bought e ]
• In some languages, “move wh” does not apply on the way from D-Str to S-Str, but “covertly” on the way from S-Str to LF: Chinese (Huang 1982).

(29) Zhangsan zhidao [ta muqin kanjian shei] (Huang 1982)
Zhangsan know his mother see who
a. “Zhangsan knows who his (=eg Zhangsan’s) mother saw.”
b. “Who does Zhangsan know his (=eg Zhangsan’s) mother saw?”

• Going back to QuNPs, in some languages, moving a QuNP by S-Str determines its scope:

(30) German: (Diesing 1997)
a. … weil ich selten jedes Cello spiele. 
   since I seldom every cello play 
   “since I seldom play every cello”  
   seldom >> every cello
b. … weil ich jedes Cello selten e spiele. 
   since I every cello seldom e play 
   “since I seldom play every cello”  
   every cello >> seldom

• The idea is that, in some other languages, like English, moving a QuNP to determine its scope happens covertly on the way from S-Str to LF. This movement operation is called “Quantifier Raising” (QR), as exemplified in (31)-(32):

(31) A student admires every professor.

(32) S-Str ⇒ LF1: [S Every professor a student admires e ]

[Quantifier Raising: syntax.

(33) Transformations: Quantifier Raising.
   [S X QuNP Y ] ⇒ [S, QuNP, [S X e, Y ] ],
   where i is an index (a natural number).

[Quantifier Raising: semantics.

(34) \[.]^{w,g}

(35) A variable assignment g is a function from expressions in the object language to objects in the world:
   In PrL: \ g: set of variables x, y, z…   → universe of individuals U
   In NatLg: \ g: set of indices 1, 2, 3, …   → universe of individuals U
   (on a trace or on a pronoun)
(36) a. In PrL: If $\alpha$ is a variable, then $[[\alpha]]^{s,g} = g(\alpha)$
b. In NatLg: If $\alpha$ is a pronoun or a trace with index i, then $[[\alpha]]^{s,g} = g(i)$

(37) In PrL:
$g^{d/v}$ reads as "the variable assignment $g'$ that is exactly like $g$ except (maybe) for $g(v)$, which equals the individual $d$".
If $\varphi$ is a formula and $v$ is a variable, then, for any world $w$,
\[
[[\forall v \varphi]]^{w,g} = 1 \quad \text{iff} \quad [[\varphi]]^{w,g^{d/v}} = 1 \quad \text{for all the } d \in D_e.
\]
\[
[[\exists v \varphi]]^{w,g} = 1 \quad \text{iff} \quad [[\varphi]]^{w,g^{d/v}} = 1 \quad \text{for some } d \in D_e.
\]

(38) In NatLg:
$g^{d/i}$ reads as "the variable assignment $g'$ that is exactly like $g$ except (maybe) for $g(i)$, which equals the individual $d$".

(39) $[[\text{[every N']}], S]]^{w,g} = 1 \quad \text{iff} \quad [[N']]^{w,g} \subseteq \{x: [[S]]^{w,gx/i}=1\}$

(40) $[[\text{[some N']}], S]]^{w,g} = 1 \quad \text{iff} \quad [[N']]^{w,g} \cap \{x: [[S]]^{w,gx/i}=1\} \neq \emptyset$

(41) $[[\text{[no N']}], S]]^{w,g} = 1 \quad \text{iff} \quad [[N']]^{w,g} \cap \{x: [[S]]^{w,gx/i}=1\} = \emptyset$

(42) $[[\text{[at most four N']}], S]]^{w,g} = 1 \quad \text{iff} \quad |[[N']]^{w,g} \cap \{x: [[S]]^{w,gx/i}=1\}| \leq 4$

(43) $[[\text{[most N']}], S]]^{w,g} = 1 \quad \text{iff} \quad |[[N']]^{w,g} \cap \{x: [[S]]^{w,gx/i}=1\}| \geq 2/3 |[[N']]^{w,g}|

(44) $[[\text{[manyprop N']}], S]]^{w,g} = 1 \quad \text{iff} \quad |[[N']]^{w,g} \cap \{x: [[S]]^{w,gx/i}=1\}| > 1/2 |[[N']]^{w,g}|

(45) $[[\text{[manycard N']}], S]]^{w,g} = 1 \quad \text{iff} \quad |[[N']]^{w,g} \cap \{x: [[S]]^{w,gx/i}=1\}| > n$

(46) …
Example 1:

(47) Some student admires every professor.

(48) S-Str \(\Rightarrow\) LF1: \(\text{[S [Every professor] some student admires } e_5]\)

(49) 

```
   q                     p
  QuNP_5  S               S
  r      u         q      p
every professor  QuNP  VP
r  u  r  u
some student admires e_5
```

(50) \([e_5]^{w,g} = g(5)\)

\([\text{admires}]^{w,g} = \{ <x,y> : x admires y in w \}\)

\([\text{admires } e_5]^{w,g} = \{ z : <z, [e_5]^{w,g}> \in [\text{admires}]^{w,g} \}\)

\([\text{admires } e_5]^{w,g} = \{ z : <z,g(5)> \in \{ <x,y> : x admires y in w \}\}\)

\([\text{admires } e_5]^{w,g} = \{ z : z admires g(5) in w \}\)

\([\text{some N'} \text{ VP}]^{w,g} = 1\)

iff \([N']^{w,g} \cap [\text{VP}]^{w,g} \neq \emptyset\) (from (4))

iff \([\text{student}]^{w,g} \cap [\text{admires } e_5]^{w,g} \neq \emptyset\)

iff \(\{z : z is a student in w\} \cap \{z : z admires g(5) in w\} \neq \emptyset\)

iff \(\{z : z is a student in w and z admires g(5) in w\} \neq \emptyset\)

\([\text{every N' s S}]^{w,g} = 1\)

iff \([N']^{w,g} \subseteq \{x : [S]^{w,g/x} = 1\}\)

iff \([\text{professor}]^{w,g} \subseteq \{x : [\text{some student admires } e_5]^{w,g/x} = 1\}\)

iff \(\{x : x is a professor in w\} \subseteq \{x : [\text{some student admires } e_5]^{w,g/x} = 1\}\)

iff \(\{x : x is a professor in w\} \subseteq \{x : z is a student in w and z admires g(5)(x) in w\} \neq \emptyset\}\)

iff \(\{x : x is a professor in w\} \subseteq \{x : z is a student in w and z admires x in w\} \neq \emptyset\}\)

That is: ‘For every x in the set of professors there is a z in the set of student admirers of x.’
Example 2:

(51) Some student admires every professor.

(52) \( \text{S-Str} \Rightarrow \text{LF1} \Rightarrow \text{LF2}: \)
\[
[S \text{ some student } [S [\text{every professor}]e_6 \text{ admires } e_5 ] ]
\]

(53)

\[
\begin{array}{c}
\text{S} \\
\text{QuNP}_6 \\
\text{some student} \\
\text{QuNP}_5 \\
\text{every professor} \\
\text{VP} \\
\text{admires}
\end{array}
\]

Exercise (to be handed in!!): Do the semantic computation of (53).