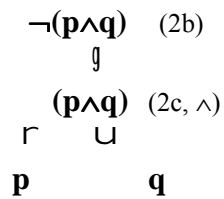


(4) Syntactic structure of $\neg(\mathbf{p}\wedge\mathbf{q})$:



QUESTION 2: Construct the syntactic tree for (5):

(5) $\neg((\mathbf{p}\wedge\mathbf{q})\rightarrow\mathbf{r}) \leftrightarrow \neg(\mathbf{s}\vee\neg\neg\mathbf{q})$

3. Semantics of PL.

■ Semantic value of the PL connectives: For any PL formulae ϕ and ψ ,

(6) Negation:

$\llbracket \neg\phi \rrbracket = 1$ iff $\llbracket \phi \rrbracket = 0$.

| ϕ | $\neg\phi$ |
|--------|------------|
| 1 | 0 |
| 0 | 1 |

(7) Conjunction:

$\llbracket \phi\wedge\psi \rrbracket = 1$ iff $\llbracket \phi \rrbracket = 1$ and $\llbracket \psi \rrbracket = 1$.

| ϕ | ψ | $\phi\wedge\psi$ |
|--------|--------|------------------|
| 1 | 1 | 1 |
| 1 | 0 | 0 |
| 0 | 1 | 0 |
| 0 | 0 | 0 |

(8) Disjunction:

$\llbracket \phi\vee\psi \rrbracket = 1$ iff $\llbracket \phi \rrbracket = 1$ or $\llbracket \psi \rrbracket = 1$

| ϕ | ψ | $\phi\vee\psi$ |
|--------|--------|----------------|
| 1 | 1 | 1 |
| 1 | 0 | 1 |
| 0 | 1 | 1 |
| 0 | 0 | 0 |

- (9) Conditional:
 $\llbracket \phi \rightarrow \psi \rrbracket = 0$ iff $\llbracket \phi \rrbracket = 1$ and $\llbracket \psi \rrbracket = 0$.

| ϕ | ψ | $\phi \rightarrow \psi$ |
|--------|--------|-------------------------|
| 1 | 1 | 1 |
| 1 | 0 | 0 |
| 0 | 1 | 1 |
| 0 | 0 | 1 |

- (10) Biconditional:
 $\llbracket \phi \leftrightarrow \psi \rrbracket = 0$ iff $\llbracket \phi \rrbracket \neq \llbracket \psi \rrbracket$.

| ϕ | ψ | $\phi \leftrightarrow \psi$ |
|--------|--------|-----------------------------|
| 1 | 1 | 1 |
| 1 | 0 | 0 |
| 0 | 1 | 0 |
| 0 | 0 | 1 |

■ Translations from natural language (English) into PL:

QUESTION 3: Translate the following sentences into PL: (mostly from GAMUT I)

- (11) a. This engine is noisy and it uses a lot of energy.
 b. Joan or Mary came.
 c. If Peter and Susan come, I will be upset.
 d. It is not the case that I will be upset if you don't come.
 e. It is not the case that Cain is guilty and Abel is not.
 f. John is not only stupid but also nasty.
 g. Johnny saw Santa and the Easter Rabbit last year, but I saw neither.
 h. Charles goes to work by car, or by bike and train.
 i. John will come only if Peter comes.
 j. Charles comes if Elsa comes, and the other way around.
 k. If father and mother both go, then I won't, but if only father goes, then I will go.

QUESTION 4: The disjunction described in (8) is called inclusive disjunction. Sometimes, though, English **or** or **either...or** is used in such a way as to exclude the possibility that both disjuncts are true, as in (12). Define another connective that captures this exclusive reading.

- (12) Either John came or Mary left.

QUESTION 5: Do the same for **only if**, exemplified in (13a), and for **unless**, illustrated in (13b). That is, give a truth table for **only-if** and for **unless**:

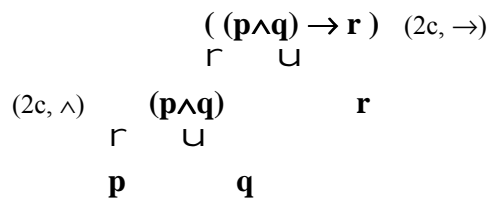
- (13) a. Only if the weather is nice will I visit Mary.
 b. Unless the weather is nice, I will visit Mary.

QUESTION 6: Show how all the five connectives can be reduced to just \neg and \vee . That is, rewrite the following expressions using exclusively \neg and \vee (and parentheses).

- $p \rightarrow q$
 $p \wedge q$
 $p \leftrightarrow q$

- Deriving the semantic value of a complex formula *compositionally*: the truth value $\llbracket \cdot \rrbracket^s$ of each node depends on the truth value of its daughter nodes and on the semantic rule corresponding to the syntactic rule through which we combined the daughters.

- (14) If Peter and Susan come, I will be upset.
 KEY: **p**=Peter comes; **q**=Susan comes; **r**=I will be upset.



| p | q | r | (p ∧ q) | ((p ∧ q) → r) |
|----------|----------|----------|----------------|----------------------|
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 0 | 1 |
| 0 | 1 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 0 | 0 | 0 | 1 |

QUESTION 7: Draw the syntactic trees and construct the truth tables for the compositional semantic interpretation of (14) and (15).

(15) It is not the case that I will be upset if you don't come.

KEY: **p**= I will be upset; **q**= you come

(16) If father and mother both go, then I won't, but if only father goes, then I will go.

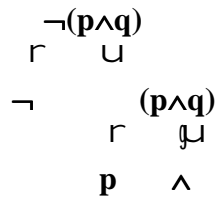
KEY: **p**=father comes; **q**=mother comes; **r**=I will go

4. Alternative Syntax and Semantics for Propositional Logic.

In sections 2 and 3, the PL connectives were treated *syncategorematically*. Syntactically, they were not lexical entries on their own, but they were each introduced into the syntactic representation by a rule. Semantically, they were interpreted contextually: we did not interpret \vee itself, but we just specified how $\phi\vee\psi$ should be interpreted once the interpretation of ϕ and ψ is given.

PL connectives can also be treated *categorematically*: we can treat them as lexical items on their own (they'll be terminal nodes in the syntactic tree, as in (4')), and, hence, we can interpret them directly: they will denote truth functions.

(4') Syntactic structure of $\neg(\mathbf{p}\wedge\mathbf{q})$:



(17) Lexical entries: the letters $\mathbf{p}, \mathbf{q}, \mathbf{r}, \mathbf{s}, \dots$, representing atomic statements, the unary connective \neg , and the binary connectives $\wedge, \vee, \rightarrow, \leftrightarrow$.

(18) Syntactic Rules:

- a. Any atomic statement is a formula in PL.
- b. If ϕ is a formula in PL, then $\neg\phi$ is a formula in PL too.
- c. If ϕ and ψ are PL formulae and ∞ is a binary connective, then $(\phi\infty\psi)$ is a PL formula.
- d. Nothing else is a formula in PL.

(19) $\llbracket \neg \rrbracket$ = the function f_{\neg} such that
 $f_{\neg}(1)=0$ and $f_{\neg}(0)=1$.

(20) $\llbracket \wedge \rrbracket$ = the function f_{\wedge} such that
 $f_{\wedge}(1,1)=1$ and $f_{\wedge}(1,0)=f_{\wedge}(0,1)=f_{\wedge}(0,0)=0$

(21) $\llbracket \vee \rrbracket$ = the function f_{\vee} such that
 $f_{\vee}(1,1)=f_{\vee}(1,0)=f_{\vee}(0,1)=1$ and $f_{\vee}(0,0)=0$

(22) $\llbracket \rightarrow \rrbracket$ = the function f_{\rightarrow} such that
 $f_{\rightarrow}(1,1)=f_{\rightarrow}(0,1)=f_{\rightarrow}(0,0)=1$ and $f_{\rightarrow}(1,0)=0$

(23) $\llbracket \leftrightarrow \rrbracket$ = the function f_{\leftrightarrow} such that
 $f_{\leftrightarrow}(1,1)=f_{\leftrightarrow}(0,0)=1$ and $f_{\leftrightarrow}(1,0)=f_{\leftrightarrow}(0,1)=0$

(24) Semantic rules:

- a. $\llbracket \neg\phi \rrbracket = \llbracket \neg \rrbracket (\llbracket \phi \rrbracket)$
- b. For any binary connective ∞ , $\llbracket \phi\infty\psi \rrbracket = \llbracket \infty \rrbracket (\llbracket \phi \rrbracket, \llbracket \psi \rrbracket)$

5. Connectives in NatLg.

Negation **not**, the coordinating conjunctions **and** and **or**, and the subordinating conjunctions **if** and **if and only if** will be treated categorically in NatLg. That is, they are entries in our lexicon and have their own denotation (as in section 4):

$$(25) \quad \llbracket \text{not} \rrbracket = \begin{array}{l} 1 \rightarrow 0 \\ 0 \rightarrow 1 \end{array}$$

$$(26) \quad \llbracket \text{and} \rrbracket = \begin{array}{l} \langle 1,1 \rangle \rightarrow 1 \\ \langle 1,0 \rangle \rightarrow 0 \\ \langle 0,1 \rangle \rightarrow 0 \\ \langle 0,0 \rangle \rightarrow 0 \end{array}$$

$$(27) \quad \llbracket \text{or} \rrbracket = \begin{array}{l} \langle 1,1 \rangle \rightarrow 1 \\ \langle 1,0 \rangle \rightarrow 1 \\ \langle 0,1 \rangle \rightarrow 1 \\ \langle 0,0 \rangle \rightarrow 0 \end{array}$$

$$(28) \quad \llbracket \text{if} \rrbracket = \begin{array}{l} \langle 1,1 \rangle \rightarrow 1 \\ \langle 1,0 \rangle \rightarrow 0 \\ \langle 0,1 \rangle \rightarrow 1 \\ \langle 0,0 \rangle \rightarrow 1 \end{array}$$

$$(29) \quad \llbracket \text{iff} \rrbracket = \begin{array}{l} \langle 1,1 \rangle \rightarrow 1 \\ \langle 1,0 \rangle \rightarrow 0 \\ \langle 0,1 \rangle \rightarrow 0 \\ \langle 0,0 \rangle \rightarrow 1 \end{array}$$

$$(30) \quad \text{Semantic rules:}$$

For any world w ,

- $\llbracket \text{not } S \rrbracket^w = \llbracket \text{not} \rrbracket^w (\llbracket S \rrbracket^w)$
- $\llbracket S_1 \text{ conj } S_2 \rrbracket^w = \llbracket \text{conj} \rrbracket^w (\llbracket S_1 \rrbracket^w, \llbracket S_2 \rrbracket^w)$

QUESTION 8: The functions denoted by the NatLg connectives above can be schönfinkelized. We do not have intuitions about what forms a constituent in PL, but we do so for English. Which syntactic structure would you assign to *if* IP_1 , IP_2 , and why? Give a schönfinkelized denotation for *if* that captures this constituency intuition.

6. Some other logical concepts.

- (31) a. A formula is a *tautology* iff it is true under any $\llbracket \cdot \rrbracket^w$. \top
 b. A formula is a *contradiction* iff it is false under any $\llbracket \cdot \rrbracket^w$. \perp
 c. A formula is *contingent* iff it is true under some $\llbracket \cdot \rrbracket^w$ and false under other $\llbracket \cdot \rrbracket^w$.
- (32) ϕ and ψ are *logically equivalent* (i.e., $\phi \leftrightarrow \psi$) iff, for every w , $\llbracket \phi \rrbracket^w = \llbracket \psi \rrbracket^w$.
 iff $\phi \leftrightarrow \psi$ is a tautology.
- (33) ψ is a *logical consequence* of ϕ (i.e., $\phi \Rightarrow \psi$)
 iff, for every world w such that $\llbracket \phi \rrbracket^w = 1$, $\llbracket \psi \rrbracket^w = 1$.
 iff $\phi \rightarrow \psi$ is a tautology.
- (34) Laws of Propositional Logic:
1. Idempotent Laws: $(\phi \vee \phi) \Leftrightarrow \phi$
 $(\phi \wedge \phi) \Leftrightarrow \phi$
 2. Commutative Laws: $(\phi \vee \psi) \Leftrightarrow (\psi \vee \phi)$
 $(\phi \wedge \psi) \Leftrightarrow (\psi \wedge \phi)$
 3. Associate Laws: $((\phi \vee \psi) \vee \pi) \Leftrightarrow (\phi \vee (\psi \vee \pi))$
 $((\phi \wedge \psi) \wedge \pi) \Leftrightarrow (\phi \wedge (\psi \wedge \pi))$
 4. Distributive Laws: $(\phi \vee (\psi \wedge \pi)) \Leftrightarrow ((\phi \vee \psi) \wedge (\phi \vee \pi)) \ominus$
 $(\phi \wedge (\psi \vee \pi)) \Leftrightarrow ((\phi \wedge \psi) \vee (\phi \wedge \pi))$
 5. Identity Laws: $(\phi \vee \perp) \Leftrightarrow \phi$
 $(\phi \vee \top) \Leftrightarrow \top$
 $(\phi \wedge \perp) \Leftrightarrow \perp$
 $(\phi \wedge \top) \Leftrightarrow \phi$
 6. Complement Laws: $(\phi \vee \neg\phi) \Leftrightarrow \top$
 $(\phi \wedge \neg\phi) \Leftrightarrow \perp$
 $\neg\neg\phi \Leftrightarrow \phi$
 7. DeMorgan's Laws: $\neg(\phi \vee \psi) \Leftrightarrow (\neg\phi \wedge \neg\psi) \ominus$
 $\neg(\phi \wedge \psi) \Leftrightarrow (\neg\phi \vee \neg\psi)$
 8. Conditional Laws: $(\phi \rightarrow \psi) \Leftrightarrow (\neg\phi \vee \psi) \ominus$
 $(\phi \rightarrow \psi) \Leftrightarrow (\neg\psi \rightarrow \neg\phi) \quad (\text{Contraposition}) \ominus$
 9. Biconditional Laws: $(\phi \leftrightarrow \psi) \Leftrightarrow (\phi \rightarrow \psi) \wedge (\psi \rightarrow \phi)$
 $(\phi \leftrightarrow \psi) \Leftrightarrow (\neg\phi \wedge \neg\psi) \vee (\phi \wedge \psi)$

QUESTION 9: Prove, using truth tables, the equivalences marked with the symbol ‘ \ominus ’ above.