Intensionality

1. Intensional Propositional Logic (IntPL).

■ Intensional PL adds some operators **O** to our standard PL. The crucial property of these operators is that, for any formula ϕ, the truth value that $[\![\mathbf{O}\phi]\!]^{w}$ yields (in the current wolrd w) depends not (just) on $[\![\phi]\!]^{w}$ but on $[\![\phi]\!]^{w'}$ for some other worlds w'. This means that the semantics of a language with an expression **O**ϕ involves *quantification over possible worlds*, which is what characterizes intensional languages.

■ Each operator **O** encodes some quantificational force and (i) a first restriction specifying the kind of worlds it quantifies over. Further restrictions on the set of worlds come from: (ii) the current evaluation world w, and –if the operator expresses an attitude (belief, desire, etc.) – from (iii) the holder of the attitude.

(1) Quantificational Force:
a. **It is necessary that** ϕ: "In *all* worlds w', $[\![\phi]\!]^{w'}=1$."
b. **It is possible that** ϕ: "In *some* world w', $[\![\phi]\!]^{w'}=1$."

(2) Restriction on situations by **O**:
a. **It is logically necessary that** ϕ: ALETHIC LOGIC
"In all *logically possible* worlds w', $[\![\phi]\!]^{w'}=1$."
b. **It is obligatory that** ϕ: DEONTIC LOGIC
"In all possible worlds w' *where all our (legal, moral, etc.) obligations are fulfilled*, $[\![\phi]\!]^{w'}=1$."
c. **It must be the case that** (as opposed to **perharps, may**, etc.) ϕ: EPISTEMIC LOGIC
"In all possible worlds w' *that conform to what we belief to be the case*, $[\![\phi]\!]^{w'}=1$."

(3) Katherine must be very nice. $\Rightarrow$ Deontic or epistemic.

(4) Restriction on worlds due to current evaluation world:
In w_1, today's chess game evolved very quickly. At 12.25pm it was clear to me that black would not win. In w_1 (at 12.15pm), the statement **I know that black will not win** is true.
In w_2, today's chess game evolved slowly. At 12.25pm it was undecided who would win. In w_2 (at 12.15pm), the statement **I know that black will not win** is false.

(5) Holder of the attitude:
a. **It is known to John that** ϕ.
b. **It is known to Peter that** ϕ.

■ To compute the semantics of IntPL formulae, we need (6) [also called "Model"]:

(6) a. a non empty set W of worlds.
b. a binary relation R in S encoding the restrictions (i), (ii) and (iii) on W, i.e., a relation R specifying which worlds are accessible from each w (and for a given attitude holder, if needed).
c. a Lexicon assigning a truth value to every propositional letter p in each world w.

1.1. Syntax of Modal PL.

(7) Lexical entries: the propositional letters **p, q, r, s**…, representing atomic statements.

(8) a. Any atomic statement --represented with the letters **p, q, r, s**,…-- is a formula in ModPL.
b. If ϕ is a formula in ModPL, then $\neg\phi$ is a formula in ModPL too.
c. If ϕ and ψ are formulae in ModPL, then $(\phi\wedge\psi)$, $(\phi\vee\psi)$, $(\phi\rightarrow\psi)$, and $(\phi\leftrightarrow\psi)$ are formulae too.
d. If ϕ is a formula in ModPL, then $\Box\phi$ and $\Diamond\phi$ are formulae in ModPL too.
e. Nothing else is a formula in ModPL.

QUESTION: Translate (9) into ModPL.

(9) a. It is possible that you don’t understand me, but it isn’t necessary.
b. If it may be raining, then it must be possible that it is raining.
c. It is possible that, if it may be raining, it is raining.
d. If it may be necessary that it is raining, then it must be raining.
e. Maybe it is raining, and perhaps this is necessary. [Two translations]

1.2. Semantics of Modal PL.

(10) For all propositional letters p and worlds w, $[[p]]^w$ is fixed in the Lexikon (values 1 or 0).

(11) For any ModPL formulae ϕ and ψ, and for any world w:
a. $[\![\neg\phi]\!]^w = 1$ iff $[\![\phi]\!]^w = 0$.
b. $[\![\phi\rightarrow\psi]\!]^w = 1$ iff $[\![\phi]\!]^w = 0$ or $[\![\psi]\!]^w = 1$.
…
c. $[\![\Box\phi]\!]^w = 1$ iff for all $w'\in W$ such that wRw': $[\![\phi]\!]^{w'} = 1$.
d. $[\![\Diamond\phi]\!]^w = 1$ iff for some $w'\in W$ such that wRw': $[\![\phi]\!]^{w'} = 1$.

QUESTION: Consider the Model in (12), where the solid arrows encode the accessibility relation $R_{Deontic}$ and the dotted arrows encode the accessibility relation $R_{Epistemic}$. Decide which of the formulae in (13) are true in w_1, which are true in w_2., and which are true in w_3.

(12)

$\neg p$ w_1 • • w_2 p

•
w_3 p

(13) a. $\Box_{Deo}\, p$
b. $\Box_{Epi}\, p$
c. $\Diamond_{Deo}\, p$
d. $\Diamond_{Epi}\, p$
e. $\Box_{Epi} \Diamond_{Deo}\, p$
f. $\Box_{Epi}\, p \rightarrow \Box_{Epi} \Box_{Deo}\, p$
g. $\Diamond_{Deo} \Box_{Epi}\, p$
h. $p \rightarrow \Box_{Epi} \Diamond_{Deo}\, p$

2. Intensional Predicate Logic (IntPrL).

- Domain of individuals, and names.
  We consider that each world w has its own domain of individuals, $D^w{}_e$. Two different worlds may have different domains of individuals, since, e.g, I, Maribel, may exist in one but fail to exist in the other.
  Hence, the denotation of a name is dependent on the evaluation w: $[[$**Maribel**$]]^w$ = me iff I exist in w. In a world w where I don't exist, $[[$**Maribel**$]]^w$ is undefined.

2.1. Syntax of Modal PrL.

- Primitive vocabulary:

(15) Lexical entries, with a denotation of their own:
a. A set of individual constants, represented with the letters **a**, **b**, **c**, **d**…
b. A set of individual variables $\mathbf{x_0}$, $\mathbf{x_1}$, $\mathbf{x_2}$,… $\mathbf{y_0}$, $\mathbf{y_1}$, $\mathbf{y_2}$,… Individual constants and individual variables together constitute the set of terms.
c. A set of predicates, each with a fixed n-arity, represented by **P**, **Q**, **R** …

(16) Symbols treated syncategorematically:
a. The PL logical connectives.
b. The quantifier symbols $\exists$ and $\forall$.
c. The intensional (modal alethic) operators $\Box$ and $\Diamond$.

- Syntactic rules:

(17) a. If P is a n-ary predicate and $t_1 \dots t_n$ are all terms, then $P(t_1 \dots t_n)$ is an atomic formula.
b. If ϕ is a formula, then $\neg\phi$ is a formula.
c. If ϕ and ψ are formulae, then $(\phi\wedge\psi)$, $(\phi\vee\psi)$, $(\phi\rightarrow\psi)$, $(\phi\leftrightarrow\psi)$ are formulae too.
d. If ϕ is a formula and v is a variable, then $\forall v\phi$, $\exists v\phi$ are formulae too.
e. If ϕ is a formula, then $\Box\phi$ and $\Diamond\phi$ are formulae too.
f. Nothing else is a formula in PrL.

2.2. Semantics of ModPrL.

■ Model:

(18) A model for a ModPrL language consists of:
a. a. a non empty set W of worlds.
b. a binary accessibility relation R in W.
c. a domain of individuals for each world, D^w_e.
d. a Lexicon assigning a denotation to every constant for each world w.
e. an assignment function g that assigns individuals to variables.

■ Semantic rules:

(19) a. If α is a constant (excluding syncategorematically treated symbols), then $[[\alpha]]^{w,g}$ is specified in the Lexikon for each w.
b. If α is a variable, then $[\![\alpha]\!]^{w,g} = g(\alpha)$

(20) a. If P is a n-ary predicate and $t_1 \ldots t_n$ are all terms, then, for any w,

$[\![P(t_1 \ldots t_n)]\!]^{w,g} = 1$ iff $[\![t_1]\!]^{w,g} \in D^w_e, \ldots, [\![t_n]\!]^{w,g} \in D^w_e$, and $<[\![t_1]\!]^{w,g}, \ldots, [\![t_n]\!]^{w,g}> \in [\![P]\!]^{w,g}$

$[\![P(t_1 \ldots t_n)]\!]^{w,g} = 0$ iff $[\![t_1]\!]^{w,g} \in D^w_e, \ldots, [\![t_n]\!]^{w,g} \in D^w_e$, and $<[\![t_1]\!]^{w,g}, \ldots, [\![t_n]\!]^{w,g}> \notin [\![P]\!]^{w,g}$

If ϕ and ψ are formulae, then, for any world w,

b. $[\![\neg\phi]\!]^{w,g} = 1$ iff $[\![\phi]\!]^{w,g} = 0$
$[\![\neg\phi]\!]^{w,g} = 0$ iff $[\![\phi]\!]^{w,g} = 1$

c. $[\![\phi\rightarrow\psi]\!]^{w,g} = 1$ iff $[\![\phi]\!]^{w,g} = 1$ and $[\![\psi]\!]^{w,g} = 1$
or $[\![\phi]\!]^{w,g} = 0$ and $[\![\psi]\!]^{w,g} = 1$
or $[\![\phi]\!]^{w,g} = 0$ and $[\![\psi]\!]^{w,g} = 0$
$[\![\phi\rightarrow\psi]\!]^{w,g} = 0$ iff $[\![\phi]\!]^{w,g} = 1$ and $[\![\psi]\!]^{w,g} = 0$.

d. $[\![\Box\phi]\!]^{w} = 1$ iff for all $w' \in W$ such that wRw': $[\![\phi]\!]^{w'} = 1$
$[\![\Box\phi]\!]^{w} = 0$ iff there is an $w' \in W$ such that wRw': $[\![\phi]\!]^{w'} = 0$

e. $[\![\Diamond\phi]\!]^{w} = 1$ iff there is an $w' \in W$ such that wRw': $[\![\phi]\!]^{w'} = 1$
$[\![\Diamond\phi]\!]^{w} = 0$ iff for all $w' \in W$ such that wRw': $[\![\phi]\!]^{w'} = 0$

f. If φ is a formula and v is a variable, then, for any world w,
$[\![\forall v\phi]\!]^{w,g} = 1$ iff $[\![\phi]\!]^{w,gd/v} = 1$ for all the $d \in D^w_e$.
$[\![\forall v\phi]\!]^{w,g} = 0$ iff $[\![\phi]\!]^{w,gd/v} = 0$ for some $d \in D^w_e$.

g. If φ is a formula and v is a variable, then, for any world w,
$[\![\exists v\phi]\!]^{w,g} = 1$ iff $[\![\phi]\!]^{w,gd/v} = 1$ for some $d \in D^w_e$
$[\![\exists v\phi]\!]^{w,g} = 0$ iff $[\![\phi]\!]^{w,gd/v} = 1$ for all $d \in D^w_e$

3. Natural Language and Intensionality.

- So far, in NatLg, we have interpreted sentences and smaller constituents with respect to one world: the evaluation world w specified in $[\![.]\!]^{w,g}$. Depending of which world we take as the evaluation world, the interpretation may differ, of course:

(21) a. $[\![$**Bush wins the elections in 2004**$]\!]^{w1,g} = 1$
b. $[\![$**Bush wins the elections in 2004**$]\!]^{w2,g} = 0$

(22) a. $[\![$**The president of the U.S. in Spring 2005**$]\!]^{w1,g} = b$ (=Bush)
b. $[\![$**The president of the U.S. in Spring 2005**$]\!]^{w2,g} = k$ (=Kerry)

(23) a. $[\![$**U.S. senator in 2005**$]\!]^{w1,g} = \{a, b, c, \ldots\}$
b. $[\![$**U.S. senator in 2005**$]\!]^{w2,g} = \{m, n, o, \ldots\}$

- Now, we will introduce intensional operators in our NatLg grammar: modal auxiliaries like **must, can, may, should, might**, etc. and sentence embedding verbs like **believe, hope, want**, etc. For the sentences (23)-(27) to be evaluated wrt to a given evaluation world w, the clauses embedded under the intensional operators will have to be evaluated with respect to worlds other than w itself.

(23) Bush **can** win the next elections.
(24) Bush **can**not win the next elections.
(25) Bush **should** win the next elections.
(26) Ann **believes** Bush will win the next elections.
(27) Ann doesn't **hope** Bush wish the next elections.

3. 1. Syntax of Intensional NatLg.

(28)
S $\rightarrow$ NPsu Pred
Pred $\rightarrow$ INFL VP
INFL $\rightarrow$ (NEG) (MOD) 3rd sing
NEG $\rightarrow$ **not**
MOD $\rightarrow$ **must, can, may, should, might**, etc.
VP $\rightarrow$ V_{intr}
VP $\rightarrow$ V_{trans} NP_{DO}
VP $\rightarrow$ V' NP_{IO}
V' $\rightarrow$ $V_{ditrans}$ NP_{DO}
VP $\rightarrow$ V_{sent} S'
S' $\rightarrow$ **that** S
V_{sent} $\rightarrow$ **believe, hope, want**, etc.
…

(29) INFL raising rule (obligatory):
$[_S\ NP_{su}\ INFL\ X] \Rightarrow [_S\ INFL\ [_S\ NP_{su}\ X]\ ]$

3.2. Semantics for Intensional NatLg.

- An intensional semantics for NatLg adds some intensional operators, as we did with □ and ◊ in ModPL and ModPrL, except that NatLg is richer and combines modalities from different Intensional PLs and PrLs: epistemic, deontic, doxastic, bouletic, etc. We will specify the kind of modality (or Modal Base or conversational background) as in (30):

(30) For any worlds w and w', and for any accessibility relation R:

a. Epistemic R: Epi.

$wEpi_x w'$ iff w' conforms to what x knows in w.[1]

b. Deontic R: Deo.

wDeow' iff all the obligations/requirements (to reach a given goal) are fulfilled in w', and w' is maximally similar to w otherwise.

c. Doxastic R: Dox.

$wDox_x w'$ iff w' conforms to what x believes in w to be the case.

d. Bouletic: Bou.

$wBou_x w'$ iff w' conforms to what x desires in w for it to be the case.

- Semantic rules for Modals: **must, can, may, should, might**, etc.

(31) $[\![\textbf{must}_{\textbf{Deo}}\ \textbf{S}]\!]^{w,g} = 1$ iff $\{w': wDeow'\} \subseteq \{w': [\![\textbf{S}]\!]^{w',g} = 1\}$
iff $\forall w'\ [\ wDeow' \rightarrow [\![\textbf{S}]\!]^{w',g} = 1\ ]$
(I.e., iff in *all* worlds w' that conform to our obligations in w, S is true is that w'.)

(32) $[\![\textbf{may}_{\textbf{Deo}}\ \textbf{S}]\!]^{w,g} = 1$ iff $\{w': wDeow'\} \cap \{w': [\![\textbf{S}]\!]^{w',g} = 1\} \neq \varnothing$
iff $\exists w'\ [\ wDeow' \wedge [\![\textbf{S}]\!]^{w',g} = 1\ ]$
(I.e., iff in *some* world w' that conforms to our obligations in w, S is true is that w'.)

(33) $[\![\textbf{must}_{\textbf{Epi}}\ \textbf{S}]\!]^{w,g} = 1$ iff $\{w': wEpiw'\} \subseteq \{w': [\![\textbf{S}]\!]^{w',g} = 1\}$
iff $\forall w'\ [\ wEpiw' \rightarrow [\![\textbf{S}]\!]^{w',g} = 1\ ]$
(I.e., iff in *all* worlds w' that conform to what is known in w, S is true is that w'.)

(34) $[\![\textbf{may}_{\textbf{Epi}}\ \textbf{S}]\!]^{w,g} = 1$ iff $\{w': wEpiw'\} \cap \{w': [\![\textbf{S}]\!]^{w',g} = 1\} \neq \varnothing$
iff $\exists w'\ [\ wEpiw' \wedge [\![\textbf{S}]\!]^{w',g} = 1\ ]$
(I.e., iff in *some* world w' that conforms to what is known in w, S is true is that w'.)

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[1] A common way to write NatLg accessibility relations is this:

(i) $w' \in Epi_x(w)$ iff w' conforms to what x knows in w.

QUESTION: Do the semantic computation of the following sentences, step by step, as usual:

(35) a. John must$_{Epi}$ be helping Mary.
b. John cannot$_{Deo}$ introduce Mary to Sue.
c. Pat may$_{Epi}$ have met Paul already.
d. Pat shouldn't$_{Deo}$ visit Ann.

EXERCISE: Take the modal **can** in (36) as deontic (= "is allowed to"). Still, the sentence in (36) has two possible readings. Your tasks are: (i) give a clear English paraphrase of those two readings and explain why the two readings are not truth-conditionally equivalent (i.e., describe a world where one reading is true and the other one is false); (ii) give the LF syntactic structure corresponding to each reading; and (iii) do the semantic computation for each reading.

(36) Every intern can$_{Deo}$ go-out-this-weekend.

■ Semantics for sentence embedding verbs: **believe, hope**, etc.

(37) Cori hopes that John wins.

(38) $[\![\, [_{S'} \textbf{ that S}] \,]\!]^{w,g} \;=\; \{w'': [\![\textbf{S}]\!]^{w'',g} = 1\}$

(39) $[\![\textbf{hope}]\!]^{w,g} \;=\; \{<x, P>: \forall w' \,[wBou_x w' \rightarrow w' \in P]\}$

(40) $[\![\, [\, \mathbf{V_{sent}} \;\; \textbf{S'}] \,]\!]^{w,g} \;=\; \{x: <x, [\![\textbf{S'}]\!]^{w,g}> \in [\![\mathbf{V_{sent}}]\!]^{w,g}\}$

■ Example:

(41)
S
e i
Cori VP
e i
hopes S'
e i
that S
e i
John **wins**

$[\![$**John**$]\!]^{w,g}$ = j

$[\![$**wins**$]\!]^{w,g}$ = {y: y wins in w}

$[\![$ **[$_S$ John wins]** $]\!]^{w,g}$ = 1 iff j $\in$ {y: y wins in w}
= 1 iff j wins in w

$[\![$ **[$_{S'}$ that [$_S$ John wins]]** $]\!]^{w,g}$ = {w": $[\![$ **[$_S$ John wins]**$]\!]^{w'',g}$ =1}
= {w": j wins in w"}

$[\![$**hopes**$]\!]^{w,g}$ = {<z, P>: $\forall$w' [wBou$_z$w' $\rightarrow$ w' $\in$ P] }

$[\![$ **hopes [$_{S'}$ that John wins]** $]\!]^{w,g}$
= { x: <x, $[\![$ **[$_{S'}$ that John wins]** $]\!]^{w,g}$> $\in$ $[\![$**hope**$]\!]^{w,g}$ }
= { x: <x, $\underline{\text{\{w'': j wins in w''\}}}$> $\in$ $\underline{\text{\{<z, P>: }\forall\text{w' [wBou}_{\underline{z}}\text{w' }\rightarrow\text{ w' }\in\text{ P]\}}}$ }
= { x: $\forall$w' [wBou$_{\underline{x}}$w' $\rightarrow$ w' $\in$ $\underline{\text{\{w'': j wins in w''\}}}$]}
= { x: $\forall$w' [wBou$_{\underline{x}}$w' $\rightarrow$ $\underline{\text{j wins in w'}}$]}

$[\![$**Cori**$]\!]^{w,g}$ = c

$[\![$ **Cori [$_{VP}$ hopes that John wins]** $]\!]^{w,g}$ = 1
iff $[\![$**Cori**$]\!]^{w,g}$ $\in$ $[\![$ **hopes [$_{S'}$ that John wins]** $]\!]^{w,g}$
iff c $\in$ { x: $\forall$w' [wBou$_x$w' $\rightarrow$ j wins in w'] }
iff $\forall$w' [wBou$_c$w' $\rightarrow$ j wins in w']

QUESTION: Give a lexical denotation for **believe**. Then, draw the syntactic tree and do semantic computation for sentence (43).

(42) $[\![$**believes**$]\!]^{w,g}$

(43) Carmen believes that John likes Steve.